The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING 2010-11

MECHANICS OF SOLIDS 2

Time allowed THREE Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer FIVE questions

Only silent, self contained calculators with a Single-Line Display, or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL:

INFORMATION FOR INVIGILATORS:

Turn over

1. The centroid of area, G, of the cross-section shown in Fig. Q1 is positioned 10.96 mm from the top edge and 20.96 mm from the left edge of the section, as shown. Its second moment of areas and product moment of area about the x-y axes, through its centroid, are:

I_x = 91,188 mm^4
I_y = 258,227 mm^4
I_{xy} = 86,818 mm^4

The section is subjected to a bending moment M_y = 1000 Nm acting in the y-direction.

Determine:

(a) the principal 2nd moments of area of the section (sketch the Mohr's circle showing points corresponding to the x and y axes and the principal axes) [6 marks]

(b) the orientation angle of the principal axes with respect to the y-axis (show on a sketch of the cross-section) [4 marks]

(c) the magnitude of the bending stress at position A [10 marks]

The co-ordinate transformation equations for a set of axes, p-q, angled θ anticlockwise to the x-y axes, are,

$$p = x\cos\theta + y\sin\theta$$

$$q = -x\sin\theta + y\cos\theta$$

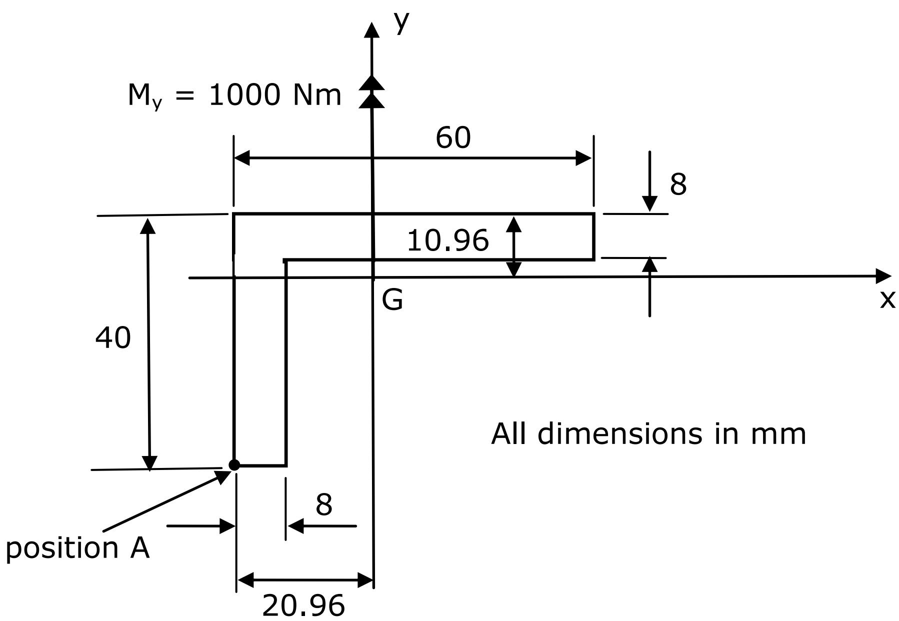


Fig. Q1

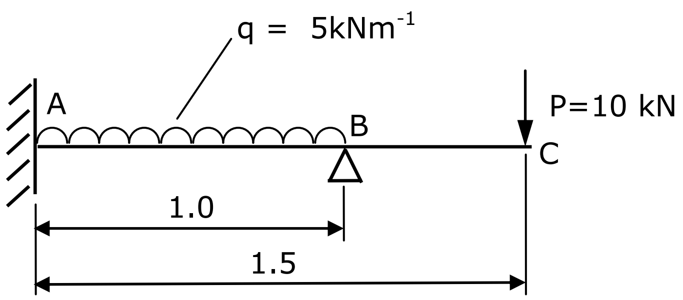
2. The propped cantilever beam, ABC, span length 1.5 m , shown in Fig Q2, is subjected to a point load 10 kN, acting at its free end, C. The prop at B is positioned 1 m from the built-in end, A. A uniformly distributed load of 5 kNm^{-1} acts over part of the span from the support, A, to the prop, B.

Use Macaulay's method to determine:

(a) The vertical reaction forces at support, A, and at prop, B. [14 marks]

(b) the magnitude of the deflection of the beam at the free end, C. [6 marks]

[Assume $EI = 10^4$ Nm^2]



All dimensions in metres

Fig Q2

3. A solid circular shaft with a diameter of 100 mm and length of 10 m, is subjected to an external torque. The material's shear modulus of elasticity, G, is 80 GPa, and the yield shear strength, τ_{yield}, is 160 MPa.
Under this loading, the cross section of the shaft has a plastic zone 30 mm deep from the surface.

By assuming an elastic-perfectly plastic material, determine:

(a) The torque acting at the cross section. [8 marks]

(b) The twist angle caused by the loading. [6 marks]

(c) The permanent twist angle when the external load is released. [6 marks]

The torsion equation for a linear-elastic circular shaft is:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

where, the polar 2nd moment of area, $J = \frac{1}{32}\pi D^4$

4. A slender half ring structure with a radius of R is supported at one end with a hinge mechanism and at the other end with a hinge-roll mechanism, as shown in Fig Q4. The structure is subjected to a vertical load P in the middle of the ring and a horizontal force Q acting at the hinge-roll support. The latter ensures that there is no displacement in the horizontal direction.

By using the strain energy method, and considering bending strain energy only, determine the horizontal force Q in terms of P. [20 marks]

The strain energy due to bending is given as:

$$U = \int \frac{M^2}{2EI} ds$$

where for a curved beam: $ds = R d\phi$

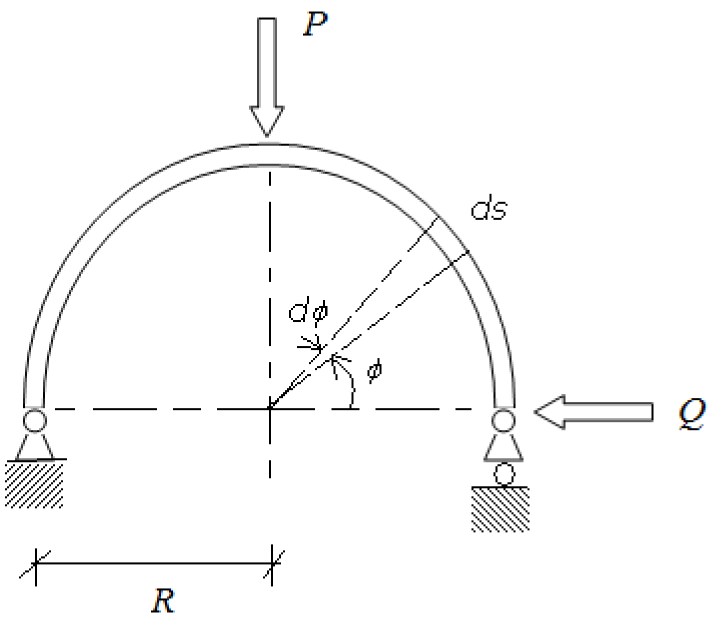


Fig Q4

5. A thin-walled beam, with the section shown in Fig Q5, is subjected to a vertical load.

(a) Determine the location of the shear centre of the beam. [15 marks]

(b) If the vertical external load is applied on the central web of the beam, as shown, describe briefly how the beam will deform. [5 marks]

The shear stress distribution in a cross section is expressed as:

$$\tau = \frac{S}{I\,z} \int_A y\,dA$$

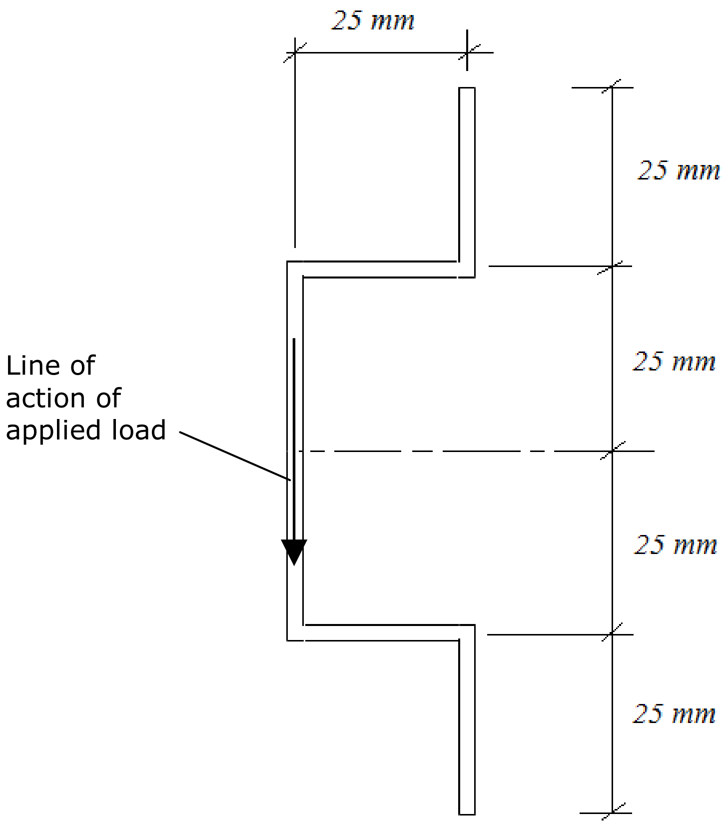


Fig Q5

6. A pin-jointed truss structure consists of two elements, AB and BC, as shown in Fig Q6. The structure is subjected to a vertical load F at point B. The cross-sectional area of each truss element is A_e and the material's modulus of elasticity is E_e. L is the vertical distance between the supports A and C.

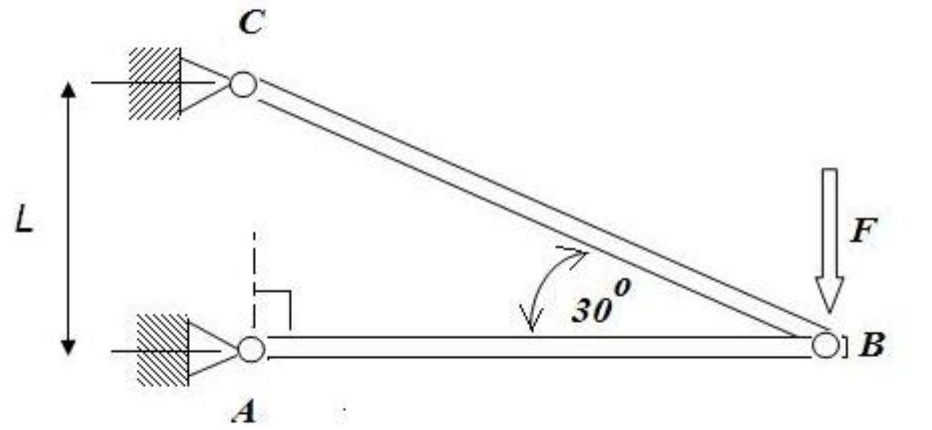


Fig Q6

(a) Construct the stiffness matrix of the structure. [7 marks]

And, using the stiffness matrix:

(b) Calculate the horizontal and vertical displacements of point B. [7 marks]
(c) Determine the reactions on the supports. [6 marks]

The stiffness matrix of a truss element is:

$$[k_e] = \left(\frac{A_e E_e}{L_e}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

where L_e is the length of the element.

7. (a) Briefly explain how fatigue cracks initiate and propagate. [8 marks]

   (b) How can a component with holes be designed for improved fatigue resistance? [2 marks]

   (c) A wide steel plate is subjected to cyclic loading over a stress range from zero stress to σ_{max}=200MPa. The steel has the following properties:

   Young's modulus, E = 207GPa
   Yield stress, σ_{yield}=630MPa
   Fracture toughness, K_{IC}=104MPa $m^{1/2}$

   It is found from experiments that the fatigue crack growth behaviour for this steel is described by the relation:

$$\frac{da}{dN} = 6.9 \times 10^{-30} (\Delta K)^{2.0}$$

   Also, Y=1.12, $K_{IC} = Y\sigma\sqrt{\pi a}$ and fatigue life, $N_f = \frac{1}{AY^2(\Delta\sigma)^2\pi} \ln\left(\frac{a_c}{a_i}\right)$.

   If the plate contains a through thickness edge crack of length 0.5mm, what will be its fatigue life?

   [10 marks]

8. A pair of mild steel cylinders of equal length have the following dimensions:

   Cylinder 1: 40mm bore and 80.1mm outside diameter
   Cylinder 2: 80mm bore and 120mm outside diameter

   i.e. there is a diametral interference of 0.1mm. The larger cylinder (Cylinder 2) is heated, placed around and allowed to shrink onto the cooled smaller cylinder (Cylinder 1).

   Calculate and plot the radial and circumferential stresses in both cylinders after assembly.

   [Assume Young's modulus, E = 200 GPa and Poisson's ratio, v = 0.3]

   For a thick cylinder under pressure:

$$\sigma_r = A - \frac{B}{r^2}$$

$$\sigma_\theta = A + \frac{B}{r^2}$$

   and

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}(\sigma_\theta - \nu\sigma_r)$$

   [20 marks]

End